Generation and annihilation of scalar particles due to a curved expanding and contracting space-time.

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DOI: 10.13140/RG.2.2.13363.37929

October 20, 2019

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Abstract

While a theory calculating cosmological generation of particles in a case of expanding space-time is quite developed, we study here the case of a space-time which is expanding and then contracting back. Our aim is not only to calculate cosmological generation of particles in case of expanding period of such a space-time, but also in the contracting period. The simplest case of fields studied in this connection is a scalar field. We will show in our paper that the quantum field has delocalized in the conformal time η particle-like modes u_k^{in} and two localized in the conformal time modes u_0^{in} and u_1^{in} for our scale factor $C(\eta) = A - B\tanh^2(\eta\rho)$ describing the space-time expanding from the value $C(\eta \to -\infty) = A$ and then contracting back to the value $C(\eta \to +\infty) = A$. Here ρ is an inverse width of the space expanding and contracting peak at $\eta = 0$. The vacuum for these states $\mid 0, out >$ defined through massive modes u_k^{out} and through modes u_0^{out} and u_1^{out} is the same as the vacuum $\mid 0, in >$. A detector shows that there are no mass particles and no localized states for $\eta \to +\infty$ for non-accelerating case. For $\eta \to -\infty$ a Minkowski space-time is realized, as it is realized also in the *out* case. We have found that for the inertial observer there are no particles detected in the vacuum $\mid 0, out >$. Let us note that in difference of the cases discussed by Birrell N. D. and Davies P. C. W. we have found that besides the mass k-particles there are present localized modes (denoted as 0 and 1) as concerning their development in the conformal time η. The quantum field has delocalized in the conformal time η particle-like modes u_k^{out} which in the -out region have k-dependent phase shifts with respect to the quantum field delocalized in the conformal time η particle-like modes u_k^{in} in the -in region. The phase shift of delocalized modes (k-particles) is due to scattering in the gravitational field leading to expansion and contraction of the space. Thus while in the expansion phase in accordance with N. D. Birrell and P. C. W. Davies there is present generation of particles, due to nonpresence of particles in $\eta \to +\infty$ conformal time it is clear that in the phase of contraction of conformal factor there is present annihilation of particles from their peak state, where they are from the generation process occurring.

Contents

1 Introduction.

In the summer term 2015 at Theoretical physics at University of Heidelberg L. Witkowski lectures [1] on "Quantum Field Theory in Curved Spacetimes" had the following topics. 1. Recall the main points of the canonical quantisation of a scalar field in Minkowski space. 2. Present how canonical quantisation is performed in curved space and focus on mode expansions and Bogolyubov transformations. 3. Discuss the *vacuum* in QFTs (Quantum Field Theories) in curved space-time. There are various definitions which are covered. From there they examine to what extent the concept of a *particle* holds in curved space-time. 4. Calculate Correlation functions and the amplitude of field fluctuations which is then useful for inflation later. 5. Our universe is currently best described as a de Sitter (dS) universe. They are discussing properties of dS space including useful coordinates, slicings etc. Describe how to quantise fields in dS and introduce the concept of the Bunch-Davies vacuum. 6. Introduce the concept of cosmological inflation. Explain how inflation solves the flatness and the horizon problem. Mention possible observables. Discuss the concept of slow-roll inflation. 7. Present the famous calculation of the primordial fluctuation spectra generated by quantum fluctuations during inflation. 8. The Unruh effect predicts that particles will be detected in a vacuum by an accelerated observer. Concepts needed include: uniformly accelerated motion, Rindler space-time in 1+1 dimensions, quantization of a mass-less scalar field in Rindler space-time, Rindler and Minkowski vacuum. The goal of this talk is to derive the density of particles and the Unruh temperature. 9. Explain Hawking's classic calculation. Arrive at the statement that Black Holes radiate. 10. Is fundamental information about the quantum state of matter undergoing gravitational collapse irretrievably lost behind the event horizon of the resulting black hole? If so, the Hawking emission from the black hole is in the form of thermal radiation, which carries little or no information about the initial quantum state of the system. If the black hole evaporates completely, that information would be lost, in violation of the rules of quantum theory. The idea of Black Hole Complementarity can show a way out of this. 11. Even if the Black Hole Information Paradox is seemingly solved, it was found recently that there are further puzzling facts. It is argued that the following three statements cannot all be true: (i) Hawking radiation is in a pure state, (ii) the information carried by the radiation is emitted from the region near the horizon, with low energy effective field theory valid beyond some microscopic distance from the horizon, and (iii) the in-falling observer encounters nothing unusual at the horizon. Perhaps the most conservative resolution is that the in-falling observer burns up at the horizon. Alternatives would seem to require novel

dynamics that nevertheless cause notable violations of semi-classical physics at macroscopic distances from the horizon. 12. The entanglement entropy is a fundamental quantity which characterizes the correlations between sub-systems in a larger quantum-mechanical system. For two sub-systems separated by a surface the entanglement entropy is proportional to the area of the surface and depends on the UV cutoff which regulates the short-distance correlations. The geometrical nature of the entanglement entropy calculation is particularly intriguing when applied to black holes when the entangling surface is the black hole horizon. 13. The axion is a proposed pseudo-scalar particle that couples to QCD. Interestingly it is possible for axions to constitute Dark Matter. Axion Dark Matter can be produced in the early universe by the misalignment mechanism. In this talk properties of axions are to be introduced and the misalignment mechanism explained. 14. There exists a conjecture for a general upper bound on the strength of gravity relative to gauge forces in quantum gravity. The bound is motivated by arguments involving the absence of black hole remnants. A sharp form of the conjecture is that there are always light *elementary* electric and magnetic objects with a mass/charge ratio smaller than the corresponding ratio for macroscopic extreme black holes, allowing extreme black holes to decay.

Quantum mechanics in a non-relativistic case [2] was generalized to the relativistic case, see [3] and [4], in a Minkowski space-time. Then generalizing the relativistic case in the Minkowski space-time lead to a natural question how can be quantum theory generalized for curved space-times. In [5] B. S. De Witt's Quantum Field Theory In Curved Space-time in an expanded version of a talk given at the meeting of the APS Division of Particles and Fields in Williamsburg, Virginia, September 1974 there is reviewed the Quantum field theory, and how it predicts a number of unusual physical effects in non-Minkowskian manifolds (flat or curved) that have no immediate analogs in Minkowski space-time. He review the following examples: (1) The Casimir effect; (2) Radiation from accelerating conductors; (3) Particle production in manifolds with horizons, including both stationary black holes and black holes formed by collapse. In the latter examples curvature couples directly to matter through the stress tensor and induces the creation of real particles. However, it also induces serious divergences in the vacuum stress. These divergences are analyzed, and methods for handling them are reviewed. In 1967 A. D. Sakharov in [6] considered the hypothesis which identifies the action with the change in the action of quantum fluctuations of the vacuum if space is curved. Thus, he consider the metrical elasticity of space as a sort of level displacement effect (emergent space-time effect). Recent development of emergent space-time is described in [7] - [9]. This problem, similar to condensed matter theory of solids, is connected with calculation of expected

values of quantum fields in vacuum and is still developing. As in solids, where there are expanding and contracting crystals under some conditions generating phonons, the generation and annihilation of scalar particles due to a curved expanding and contracting space-time is an interesting problem.

In [10] N. D. Birrell and P.C.W. Davies study the cosmological creation of particles. Calculation of expected values of quantum fields in vacuum is important and here in this paper we use the way of [10] to calculate cosmological generation of particles in some special case. The fact that quantum fluctuations of the inflaton field can back-react on space-time and form classical inhomogeneities, giving rise to CMB anisotropies and big structures like galaxies and clusters, is a property of Quantum Field Theory in curved space, and one of the great successes of inflation [11]. However authors of [12] have studied the evolution of a massive, non-interacting and non-minimally coupled scalar field in the exponentially expanding de Sitter space, which has relevance for the early and late time Universe. The Bunch-Davies vacuum state in the de Sitter space-time has the de Sitter invariance of this state which is independent of the details of renormalization and can be understood as a manifestation of covariant conservation. The behavior of the modes in the Bunch-Davies state can be interpreted to constantly go through a particle creation process as indicated by a nontrivial Bogolubov transformation. However this is not visible in the semi-classical back-reaction: it bears no sign of a density from classical particles and implies strictly w = -1 for the equation of state. Then there are in [10] interesting results to calculate cosmological generation of particles in case of expanding space-time using Quantum Field Theory in Curved Space-times. On dimensional grounds it is generally believed that quantum effects of gravity should be important at least when the space-time curvature becomes comparable to the Planck length $(\frac{\hbar G}{c^3})^{\frac{1}{2}} \approx 10^{-33} cm$. For less extreme space-time curvature, one hopes that the semi-classical approximation will be valid at least in many situations. This later case of semi-classical approximation assumption to be valid will be used in our paper. From [10] study the cosmological creation of particles in case of expanding space-time from the vacuum state in conformal time $\eta \to -\infty$ one may expect that there exists the cosmological annihilation of particles in case of contracting space-time to the same vacuum state as the vacuum state in conformal time $\eta \to -\infty$. Note that in [13] Penrose R. discusses conformal structures and conformal infinities.

We study here the case of space-time which is expanding and then contracting back to the same state. Our aim is not only calculate cosmological generation of particles in case of expanding period of such a space-time, but also to study what happens in the contracting period. The simplest case of fields studied is a scalar field. This field will be considered in this paper.

2 Cosmological generation and annihilation of particles: in- and out- space-time Minkowski.

As in [10] we consider a 2-dimensional case, in which the coordinates are (t, x), of the Robertson-Walker space-time with the metric tensor given in the element ds:

$$ds^2 = dt^2 - a(t)^2 dx^2. \tag{1}$$

In standard stationary cosmological model of the spatial expansion there is a time dependent space $a(t)$ factor which is described by the form $a_{scm}(t) = a_{scm}(t_0)\exp(H(t-t_0))$ [14]. The scale factor $C(\eta) \equiv a^2(\eta)$ has an exponential form too. Here H is the Hubble constant, and η is a conformal time defined below, it is dependent on time t.

In our case the space sections of the space homogeneously expanding (or contracting) are in correspondence with the time dependent space $a(t)$ factor in (1). Let us define conformal time η by $d\eta = \frac{dt}{a}$. Then the (1) has the form:

$$ds^2 = a^2(t)(d\eta^2 - dx^2) = C(\eta)(d\eta^2 - dx^2). \tag{2}$$

where $C(\eta) \equiv a^2(\eta)$ is a conformal scale. Our aim is to study influence of the conformal scale in the form describing a curved expanding and contracting space-time:

$$C(\eta) = A - B\tanh^2(\rho.\eta). \tag{3}$$

Here A and B are constants, we will consider possible values of these constants such that the conformal scale $C(\eta) > 0$. We assume that the constant ρ is positive. It describes the width of the peak in the scale factor $C(\eta\rho)$, which is located at the conformal time $\eta = 0$. From (3) we can see that:

$$C(\eta\rho) \to A - B \tag{4}$$

for $\eta \to \pm\infty$. We will consider the cases $A > B$ and $B > 0$, e.i. the conformal scale factor is positive in (3), and describes space-time which is smoothly expanding and smoothly contracting.

As in [10] we will consider a scalar field $\phi(x)$, here $x = (t, \mathbf{x})$ with $\mathbf{x}$ vector of space coordinates, with nonzero mass which is minimally coupled ($\xi = 0$), and study cosmological generation and annihilation of modes of this field. Note that in the dimension of space-time $n = 2$ the conformal coupling ($\xi = \frac{1}{4}[\frac{(n-2)}{(n-1)}]$) and minimal coupling to the Ricci scalar R invariant are the same, they have value 0. In general this coupling term has the form:

$$(-\xi R)\phi^2. \tag{5}$$

From the variation principle we find the general equation for the scalar field ϕ:

$$[\Box + m^2 + \xi R]\phi(x) = 0. \tag{6}$$

Here the operator $\Box$ is given by:

$$\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu. \tag{7}$$

Let us take the full set $u_i(x)$ of normal solutions of the equation (6) which is orthogonal in the sense of the scalar product defined as:

$$(\phi_1, \phi_2) = -i \int_\Sigma \phi_1(x)\overleftrightarrow{\partial_\mu}\phi_2^\star(x)[-g_\sigma(x)]^{\frac{1}{2}} d\Sigma^\mu. \tag{8}$$

Here $d\Sigma^\mu = n^\mu d\Sigma$ where n^μ is a time-like vector orthogonal to a space-like hyper-surface Σ , which has unit amplitude. Hyper-surface is chosen to be a surface of the Cauchy type in the space-time. Note that this scalar product does not depend on the hyper-surface Σ. The orthogonality of the full set of solutions $u_i(x)$ (note that it will be denoted as u_i in the future) may be expressed now as:

$$\begin{aligned} (u_i, u_j) &= \delta_{ij}, \\ (u_i^\star, u_j^\star) &= -\delta_{ij}, \\ (u_i, u_j^\star) &= 0. \end{aligned} \tag{9}$$

The index i goes through all states (modes) of the orthogonal full set of solutions $u_i(x)$. The field $\phi(x)$ may be expressed through modes u_i and $u_i^\star$ as:

$$\phi(x) = \sum_i [a_i u_i(x) + a_i^\dagger u_i^\star(x)]. \tag{10}$$

Here $\dagger$ denotes conjugation of the operator $a_i^\dagger$ to the operator a_i.

Now the covariant quantisation in this theory is based on the relations:

$$[a_i, a_j^\dagger] = \delta_{ij}, \tag{11}$$

for all i, j. Here i and j indices are denoting all (normal) modes and δ_{ij} is the Kronecker delta and bracets $[..., ...]$ usual known bracets.

The vacuum state $\mid 0 >$ in the Fock space-time is defined as:

$$a_i \mid 0 >= 0 \tag{12}$$

for every i state. In a curved space-time, where the Poincare group is not the group of symmetry, Killing vectors do not exist, and thus we are not able to define time-like solutions. We will not consider some special

cases where these Killing vectors it is possible to define. In the general case there are not privileged coordinate systems which would play the role in the quantum field theory which they play in the Minkowski space-time. Thus in the general case there is impossible to find for the wave function of the field $\phi(x)$ an expression through modes u_i and $u_i^\star$ as in (10). This corresponds to a general idea of the general theory of relativity that the choose of the coordinate system is not important from the physical point of view.

Let us consider another orthonormal set of solutions $\overline{u_j(x)}$. The field $\phi(x)$ may be again developed according to this orthonormal set of solutions:

$$\phi(x) = \sum_i [\overline{a_i}\overline{u_i(x)} + \overline{a_i^\dagger}\overline{u_i^\star(x)}]. \tag{13}$$

The vacuum state $\mid \overline{0} >$ in the Fock space-time for this development of the field $\phi(x)$ is defined as:

$$\overline{a_i} \mid \overline{0} >= 0 \tag{14}$$

for every i state. There is defined a new Fock space for the development (13) of the field $\phi(x)$. Due to the fact that both solutions are the full set of solutions of the equation, it is possible to express modes $\overline{u_i}$ through the first set of solutions:

$$\overline{a_i} = \sum_j \alpha_{ji} u_i + \beta_{ji} u_i^\star. \tag{15}$$

and

$$a_i^\star = \sum_j \alpha_{ji}^\star \overline{u_i} - \beta_{ji}^\star \overline{u_i^\star}. \tag{16}$$

Bogoljubov coefficients α_{ij} and β_{ij} are expressed as:

$$\alpha_{ij} = (\overline{u_i}, u_j) \tag{17}$$

and

$$\beta_{ij} = (\overline{u_i}, u_j^\star).$$

Two Fock spaces based on two different sets u_i and $\overline{u_i^\star}$ are different if $\beta_{ij} \neq 0$. In this case:

$$a_i \mid 0 >= \Sigma_j \beta_{ji}^\star \mid \overline{1_j} > \neq 0. \tag{18}$$

Here $\mid \overline{1_j} > \equiv a_j^\dagger \mid \overline{0} >$. Number $N_i = a_i^\dagger a_i$ of particles of the i-th type in its mean value has the form has the form:

$$< \overline{0} \mid N_i \mid \overline{0} >=< \overline{0} \mid a_i^\dagger a_i \mid \overline{0} = \Sigma_j \mid \beta_{ji} \mid^2 . \tag{19}$$

This is the number of particles of modes u_i in the vacuum of modes $\overline{u_j}$.

3 Quantisation of the scalar field.

Density L of the Lagrangian is:

$$L = \frac{1}{2}[-g(x)]^{\frac{1}{2}} g^{\mu\nu}(x)\phi(x)_{,\mu}\phi(x)_{,\nu} - [m^2 - \xi R(x)]\phi^2(x), \tag{20}$$

here $\phi(x)$ is a scalar field, m is a mass of this field. There exist term $\xi R\phi^2$ which describes an interaction of the scalar field with the gravitation field through Ricci scalar $R(x)$. The ξ is a constant. The action S has the form:

$$S = \int L d^n x. \tag{21}$$

Here a point $(t, \mathbf{x})$ of n-dimensional space-time with t time and with $\mathbf{x}$ space coordinates is denoted by x. In case of two dimensional space-time we will use coordinates (t, x). Here the x denotation is the vector $\mathbf{x}$ in this two dimensional space-time. While using denotation x in the general case as above, it will be clearly identified in the two dimensional space-time denotations of coordinates (t, x) with the same x. Going to the two dimensional case in the next sessions we will not use the general denotation of coordinates. Equation for the scalar field $\phi(x)$ is obtained from the condition of zero for the first variation of the action S:

$$\delta S = 0. \tag{22}$$

The equation for the scalar field $\phi(x)$ is:

$$[\Box + m^2 + \xi R(x)]\phi(x) = 0. \tag{23}$$

Here the operator $\Box$ is defined as:

$$\Box\phi(x) = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi(x). \tag{24}$$

The parameter ξ has the value $\xi = 0$ corresponding to the minimal coupling, and the value $\xi = \frac{1}{4}[\frac{(n-2)}{(n-1)}]$ for a conformal coupling, in our case $n = 2$ and $\xi = 0$.

4 Cosmological generation and annihilation of scalar particles.

As in [10] for the two dimensional space-time the factor $C(\eta)$ is not dependent on the spatial coordinate x, spatial translations are symmetry transformation. To discuss cosmological generation and annihilation of particles in a

space-time, which is in -out and -in regions a Minkowski space-time, we will consider an example of 2d Robertson-Walker space-time in which an element of length ds^2 is given by:

$$ds^2 = dt^2 - a^2(t)dx^2. \tag{25}$$

Here sections of space are homogeneously increasing in volume (expanding) or decreasing in volume (shrinking). This increasing or decreasing depends on the function $a(t)$, which is a scalar function. Let us define the conformal time by the relation $d\eta = \frac{dt}{a(t)}$. Then the equation (25) may be rewritten in the form:

$$ds^2 = a^2(\eta)(d\eta^2 - dx^2) = C(\eta)(d\eta^2 - dx^2). \tag{26}$$

We introduced here the conformal scale factor $C(\eta)$. As we can see the element of the (space-time) length has explicitly the conformaly flat form. Note that there is a relation between time t and the conformal time η:

$$t = \int^t dt' = \int^\eta a^2(\eta')d\eta'. \tag{27}$$

The conformal scale factor $C(\eta)$ will be assumed such that in far past and far future space-time is flat, is of Minkowski type. There will be expansion, then maximum of the volume of the space and then shrinkage of the volume of the space.The conformal scale factor $C(\eta)$ we choose is:

$$C(\eta) = A - B\tanh^2(\eta\rho). \tag{28}$$

where $\rho > 0$ is the width of expansion and contraction peak in the scale factor $C(\eta)$. As we can see from (28) for conformal time going to far past $\eta \to -\infty$ we obtain $C(\eta) \to A - B$, and for conformal time going to far future $\eta \to +\infty$ we obtain $C(\eta) \to A - B$. From (26) we obtain that $A - B > 0$. Let us note that the Minkowski space-time is expected to be the same for conformal time going to far past $\eta \to -\infty$ and for conformal time going to far future $\eta \to +\infty$. We are interested now which modes will exist in conformal times in between far past and far future.

5 Modes of the scalar field which exist in conformal times in between far past and far future.

As in [10], we see that the conformal scale factor does not depend on the space coordinate. Thus space transformations are symmetry transformations in

this space-time. Note that it is the space-time in 2d in which conformal and minimal coupling are equivalent, see also [10] for discussion of conformal and minimal couplings. In scalar modes in (10) there is dependence on conformal time and on the space coordinate segregated:

$$u_k(\eta, x) = \frac{1}{\sqrt{(2\pi)}} \exp(ikx)\chi_k(\eta). \tag{29}$$

From (23) and (29) we obtain the equation for $\chi_k(\eta)$:

$$[\frac{d^2}{d\eta^2} + (k^2 + C(\eta)m^2)]\chi_k(\eta) = 0 \tag{30}$$

taking into account that $\xi = 0$ and that we have metrics (26). This equation (30) we are able to solve exactly. Besides delocalized modes (corresponding to massive particles) we will find localized modes.

6 Solution of the equation (30).

The equation (30) in its explicit form taking into account the scale factor form (28) is:

$$[\frac{d^2}{d\eta^2} + (k^2 + Am^2 - Bm^2 \tanh^2(\eta\rho))]\chi_k(\eta) = 0. \tag{31}$$

Let us introduce a dimensionless variable $\delta \equiv \eta.\rho$. Note that $\frac{d}{d\eta} = \rho.\frac{d}{d\delta}$ and that $\frac{d^2}{d\eta^2} = \rho^2.\frac{d^2}{d\delta^2}$. Now the equation (31) takes the form:

$$[-\frac{1}{2}\frac{d^2}{d\delta^2} - \frac{1}{2}(\frac{k^2}{\rho^2} + \frac{Am^2}{\rho^2} - \frac{Bm^2}{\rho^2}\tanh^2(\delta))]\chi_k(\delta) = 0. \tag{32}$$

This type of equation was solved in [15] where the corresponding equation has the form:

$$-\frac{1}{2}\frac{d^2}{d\delta^2}\chi_k(\delta) + (3\tanh^2(\delta) - 1)\chi_k(\delta) = \frac{\omega_k^2}{M^2}\chi_k(\delta). \tag{33}$$

To identify both equations (32) and (33) we have to put:

$$\frac{1}{2}\frac{Bm^2}{\rho^2} \equiv 3, \tag{34}$$

and

$$+1 + \frac{\omega_k^2}{M^2} \equiv \frac{1}{2}(\frac{k^2}{\rho^2} + \frac{Am^2}{\rho^2}),$$

e.i.:

$$m^2 = \frac{6\rho^2}{B}, \tag{35}$$

$$\frac{\omega_k^2}{M^2} = -1 + \frac{1}{2}(\frac{k^2}{\rho^2} + \frac{3A}{B}) =$$
$$= \frac{1}{2}(\frac{k^2}{\rho^2} + \frac{3A - B}{B}).$$

In the next sections we describe delocalized and localized normal modes for the equation (32) using (35).

7 Localized modes.

Using results from [15] and equations from the previous section we find that there are two localized modes. The first one, we will denote it as 0 mode, has its zero energy:

$$\omega_0^2 = 0, \tag{36}$$

and its wave function is:

$$\chi_0(\delta) = \frac{1}{\cosh^2(\delta)}. \tag{37}$$

The corresponding value of the k_0 is found from (35):

$$k_0^2 = -\frac{3A - B}{B}\rho^2. \tag{38}$$

As we can see from (38) the momentum k_0 is imaginary, it is given by:

$$k_0 = \pm i\sqrt{\frac{3A - B}{B}\rho^2} \equiv \pm iK_0. \tag{39}$$

Here we define K_0 and assume that $K_0 > 0$. Let us write the wave function for the 0 mode from (29) and (39). It has either a form:

$$u_{k_0,+}(\eta, x) = \frac{1}{\sqrt{(2\pi)}} \exp(iK_0 x)\chi_k(\eta) = \frac{1}{\sqrt{(2\pi)}}\chi_k(\eta)(A_+ \exp(+K_0 x) \tag{40}$$

or a form:

$$u_{k_0,-}(\eta, x) = \frac{1}{\sqrt{(2\pi)}} \exp(ik_0 x)\chi_k(\eta) = \frac{1}{\sqrt{(2\pi)}}\chi_k(\eta)(A_- \exp(-K_0 x). \tag{41}$$

Here A_+ and A_- are integration constants. In x space the mode 0 cannot be infinitely increasing on its boundaries $x \to \pm\infty$. Thus the wave functions (40) and (41) has to be chosen in such a way as to equal at some point x_0 defined by $u_{k_0,-}(\eta, x_0) = u_{k_0,+}(\eta, x_0)$, e.i. where:

$$A_- \exp(-K_0 x_0) = (A_+ \exp(K_0 x_0). \tag{42}$$

Thus the point x_0 is given by $\exp(2K_0 x_0) = \frac{A_-}{A_+}$. Note that the resulting function has in the point x_0 discontinuous derivative due to the fact that $K_0 \neq 0$. From this and from the condition that x_0 is real we have that the constants A_+ and A_- are either positive either negative, both simultaneously. Then the value of the point x_0 lies in the interval $-\infty < x_0 < +\infty$. The case in which the signs of the constants A_+ and A_- are different does not correspond to a solution for which in x space the mode 0 cannot be infinitely increasing on its boundaries $x \to \pm\infty$. We see that the localized in the conformal time η mode is localized also in the x coordinate with a peak at the point x_0. As we can see there is no oscillation in the conformal time η for this mode. Conformal time development is described by the function $\frac{1}{\cosh^2(\delta)}$. Note that the mode 0 for $\delta \to \pm\infty$ has its wave function with vanishing value. For $\delta \to 0$ has its wave function with maximum value 1. Thus it represents a mode which is of a localized nature in η and x coordinates. While the peak in the first variable is at the peak of the scale factor, e.i. of the expansion of space when it turns to shrinking phase from the expansion phase, the peak in the second variable x is at the point x_0 which in our case an arbitrary quantity.

The second one localized mode, we will denote it as 1 mode, has its energy:

$$\frac{\omega_1^2}{M^2} = \frac{3}{2}, \tag{43}$$

and its wave function is:

$$\chi_1(\delta) = \frac{\sinh(\delta)}{\cosh^2(\delta)}. \tag{44}$$

The corresponding value of the k_1 is:

$$k_1^2 = -\frac{3A - 4B}{B}\rho^2 = -\frac{3(A - B) - B}{B}\rho^2. \tag{45}$$

There are two cases for k_1 values. For $A > \frac{4B}{3}$ k_1^2 is negative, and thus k_1 is imaginary. For $\frac{4B}{3} > A > B > 0$ k_1^2 is positive, and thus k_1 is real.

Let us discuss the first case. As we can see from (45) the momentum k_1 is imaginary, it is given by:

$$k_1 = \pm i\sqrt{\frac{3(A-B)-B}{B}\rho^2} \equiv \pm iK_1. \tag{46}$$

Here we assume that $K_1 > 0$, Let us write the wave function for the 1 mode from (29) and (46). It has either a form:

$$u_{k_1,+}(\eta,x) = \frac{1}{\sqrt(2\pi)}\exp(iK_1x)\chi_k(\eta) = \frac{1}{\sqrt(2\pi)}\chi_k(\eta)(B_+\exp(+K_1x) \tag{47}$$

or a form:

$$u_{k_0,-}(\eta,x) = \frac{1}{\sqrt(2\pi)}\exp(iK_1x)\chi_k(\eta) = \frac{1}{\sqrt(2\pi)}\chi_k(\eta)(B_-\exp(-K_1x) \tag{48}$$

Here B_+ and B_- are integration constants. In x space the mode 1 cannot be infinitely increasing on its boundaries $x \to \pm\infty$ again, as the mode 0, for this case of imaginary k_1. Thus the wave functions (47) and (48) has to be chosen in such a way as to equal at some point x_1 defined by $u_{k_1,-}(\eta,x_1) = u_{k_1,+}(\eta,x_1)$, e.i. where:

$$B_-\exp(-K_1x_1) = B_+\exp(K_1x_1). \tag{49}$$

The point x_1 is given by $\exp(2K_1x_1) = \frac{B_-}{B_+}$. From this and from the condition that x_1 is real we have that the constants B_+ and B_- are either positive either negative, both simultaneously. Then the value of the point x_1 is in the interval $-\infty < x_1 < +\infty$. The case in which the signs of the constants B_+ and B_- is different does not correspond to a solution for which in x space the mode 1 cannot be infinitely increasing on its boundaries $x \to \pm\infty$. We see that the localized in the conformal time η mode 1 is localized also in the x coordinate with a peak at the point x_1 in this case. As we can see there is no oscillation in the conformal time η for this mode with nondecreasing amplitude, oscillations of this mode in the conformal time η have the energy given from (43). However the amplitude of oscillations is given by the function $\chi_1(\delta) = \frac{\sinh(\delta)}{\cosh^2(\delta)}$. This amplitude has its maximum for η_{max} and has its minimum for η_{min} (both have the same absolute value, there is and inflecton point in $\eta = 0$). These points can be found from the $\tanh^2(\eta_{min},\eta_{max}) = \frac{1}{2}$ Note that the points x_0 from above and the point x_1 are in general different. Conformal time development as described by the function $\frac{\sinh(\delta)}{\cosh^2(\delta)}$ for the mode 1 and for $\delta \to \pm\infty$ has its wave function with vanishing value. For $\delta \to 0$ has its wave function value 0, it is antisymmetric

around this point. Thus it represents a mode which is of a localized nature in η and x coordinates. There are two peaks (maximum and minimum) in the first variable and one peak in the second variable x, it is at the point x_1 which in our case is arbitrary.

The second case we discuss now. For $\frac{4B}{3} > A > B > 0$ k_1^2 is positive, and thus k_1 is real. Let us write the wave function for the 1 mode in this case:

$$u_k(\eta, x) = \frac{1}{\sqrt{(2\pi)}} \exp(ik_1 x)\chi_k(\eta) = \tag{50}$$

$$= \frac{1}{\sqrt{(2\pi)}} \exp(i \pm \sqrt{\frac{3(A-B)-B}{B}\rho^2} x) \frac{\sinh(\delta)}{\cosh^2(\delta)}.$$

As we can see from (45) the momentum $k_1^{'}$ is real, it is given by:

$$k_1^{'} = \pm \sqrt{\frac{3(A-B)-B}{B}\rho^2} \equiv \pm K_1^{'}. \tag{51}$$

Here we assume that $K_1^{'} > 0$. Let us write the wave function for the 1 mode from (29) and (46). It has either a form:

$$u_{k_1^{'},+}(\eta, x) = \frac{1}{\sqrt{(2\pi)}} \exp(iK_1^{'}x)\chi_k(\eta) = \frac{1}{\sqrt{(2\pi)}}\chi_k(\eta)(B_+^{'} \exp(+iK_1^{'}x) \tag{52}$$

or a form:

$$u_{k_1^{'},-}(\eta, x) = \frac{1}{\sqrt{(2\pi)}} \exp(iK_1^{'}x)\chi_k(\eta) = \frac{1}{\sqrt{(2\pi)}}\chi_k(\eta)(B_-^{'} \exp(-iK_1^{'}x). \tag{53}$$

Here $B_+^{'}$ and $B_-^{'}$ are integration constants. In x space the mode 1 is in this case oscillatory. Thus the wave functions (52) and (53) has to be chosen in such a way as to equal to some combination of functions $u_{k_1^{'},-}(\eta, x)$ and $u_{k_1,+}(\eta, x)$. The coefficients may be written in the form $B_+^{'} = B^{'} \sin(\epsilon)$ and $B_-^{'} = B^{'} \cos(\epsilon)$. The variable ϵ defines a point $x_1^{'}$, which is a shift of the variable x . From this and from the condition that $x_1^{'}$ (an arbitrary number) is a real number we have that the constants $B_+^{'}$ and $B_-^{'}$ are both real numbers. Then the value of the point $x_1^{'}$ is in the interval $-\infty < x_1^{'} < +\infty$. We see that the localized (with oscillating amplitude due to its energy $\frac{\omega_1^2}{M^2} = \frac{3}{2}$) in the conformal time η mode 1 is not localized in the x coordinate. Note that the points x_1 and $x_1^{'}$ are in general different. Conformal time amplitude development is described by the function $\frac{\sinh(\delta)}{\cosh^2(\delta)}$. Note that the mode 1 for $\delta \to \pm\infty$ has its wave function with vanishing value. For $\delta \to 0$ has its wave

function value 0, it is antisymmetric in η. Thus it represent a mode which is of a localized nature in η coordinate and delocalized in x coordinate. This type of scalar field component does not have frequency dependent on the momentum as the excitations delocalized in conformal time, as we will see in the following section. Note that the mode 1 for $\delta \to \pm\infty$ has its wave function also with vanishing value.

We can say that localized modes are localized in the $\delta = \eta\rho$ variable, e.i. in the η variable which is a conformal time.

8 Delocalized modes - scalar particles.

Besides two localized modes described above there are delocalized (in conformal time η) and x coordinates. These modes are indexed by the momentum k, we will denote them as k modes. They have energy ω_k from the solution of the equation (33):

$$\frac{\omega_k^2}{M^2} = \frac{q^2}{2} + 2 = \frac{1}{2}(\frac{k^2}{\rho^2} + \frac{3A - B}{B}). \tag{54}$$

Here we have to find dependence of the momentum q (from the solution of the equation (33) according to [15]) on the momentum k. This dependence is found from the equation of both expressions for the energy of k states:

$$\frac{q^2}{2} + 2 = \frac{k^2}{2\rho^2} + \frac{3A - B}{B}. \tag{55}$$

We find that:

$$q^2 = \frac{k^2}{\rho^2} - \frac{6A - 2B}{B} - 4. \tag{56}$$

Then k modes have energy ω_k:

$$\frac{\omega_k^2}{M^2} = \frac{q^2}{2} + 2 = \frac{k^2}{2\rho} - \frac{3A - B}{B} - 2 + 2 = \tag{57}$$

$$= \frac{k^2}{2\rho} + \frac{3A - B}{B}.$$

Thus as we can see there is the momentum k which can be find from $\frac{k^2 M^2}{2\rho}$ above. The second expression on the right hand side (57) is positive. We can define as a mass squared the positive quantity $M^2\frac{3A-B}{B}$. The mass of the k mode (we will speak about a particle) is real because the squared mass can be written as $M^2\frac{A-B}{B} + M^2\frac{2A}{B}$, this quantity is a positive quantity due to the

fact that $A - B > 0$ and $B > 0$ (and thus $A > 0$). The wave function of the k mode is:

$$\chi_k(\delta) = \exp(ik\rho\eta)(3\tanh^2(\eta\rho) - 1 - (\frac{k^2}{\rho} + 6\frac{A-B}{B}) - \tag{58}$$

$$-3isqrt(\frac{k^2}{\rho} + 6\frac{A-B}{B})\tanh(\eta\rho)).$$

For the conformal time $\eta \to +\infty$ we obtain from (58) that the wave function has a phase shift:

$$\chi_k(\delta) \to \exp(ik\rho\eta \pm \frac{i}{2}\Delta(q)), \tag{59}$$

where the q is given from (56). The phase shift $\Delta(q)$ is given by:

$$\Delta(q) = -2\arctan(\frac{3q}{2-q^2}). \tag{60}$$

As we can see there is an oscillation type of development in the conformal time η with the frequency ω_k and an oscillation type of development in the space coordinate x with the momentum k for these (delocalized solutions) particles. Conformal time development is further described by a factor $(3\tanh^2(\eta\rho) - 1 - (\frac{k^2}{\rho} + 6\frac{A-B}{B}) - 3i\sqrt{(}\frac{k^2}{\rho} + 6\frac{A-B}{B})\tanh(\eta\rho))$.

9 Discussion.

Let us now calculate in the style as in [10] the ω_{in} and ω_{out} frequencies from above. We find:

$$\omega_{in} = \omega_{out} = M\sqrt{\frac{k^2}{2\rho^2} + \frac{3A-B}{B}}. \tag{61}$$

The frequencies $\omega_\pm$ which are found from -in and -out frequencies are given by:

$$\omega_+ = \omega_{in} = \omega_{out} \tag{62}$$

and

$$\omega_- = 0. \tag{63}$$

Note that the mass of particles generated in k-modes is $M.\sqrt{\frac{3A-B}{B}} = M.\sqrt{\frac{(A-B)+2A}{B}}$. The expression below the square root is positive: $A - B > 0$ and $B > 0$. There are two localized modes. The first one,the 0 mode, has its zero energy.

In difference to expanding case [10] of the space-time, the wave functions u_k^{in} and u_k^{out} are identical besides the phase factor in u_k^{out}. As is known from integrable systems the phase factor in scattered wave function enables us to calculate the potential of scattering, see [16] and [17], where it was discussed the Inverse Scattering Method, as a methodological paper with an example - soliton (kink) solution of the Sine-Gordon Equation and a solution of a breather type.

While in general we can express the in-function u_k^{in} through u_k^{out} as:

$$u_k^{in} = \alpha_k u_k^{out} + \beta_k u_{-k}^{out*}, \tag{64}$$

where the coefficients α_k and β_k are expressed as:

$$\alpha_k = \sqrt{\frac{\omega_{out}}{\omega_{in}}} \frac{\Gamma(1 - \frac{i\omega_{in}}{\rho})\Gamma(1 - \frac{i\omega_{out}}{\rho})}{\Gamma(1 - \frac{i\omega_{+}}{\rho})\Gamma(1 - \frac{i\omega_{+}}{\rho})}, \tag{65}$$

and

$$\beta_k = \sqrt{\frac{\omega_{out}}{\omega_{in}}} \frac{\Gamma(1 - \frac{i\omega_{in}}{\rho})\Gamma(\frac{i\omega_{out}}{\rho})}{\Gamma(\frac{i\omega_{-}}{\rho})\Gamma(1 + \frac{i\omega_{-}}{\rho})}, \tag{66}$$

from (61) - (63) we find:

$$\mid \alpha_k \mid^2 = \frac{\sinh^2(\frac{\pi\omega_+}{\rho})}{\sinh(\frac{\pi\omega_{in}}{\rho})\sinh(\frac{\pi\omega_{out}}{\rho})} = 1, \tag{67}$$

and

$$\mid \beta_k \mid^2 = \frac{\sinh^2(\frac{\pi\omega_-}{\rho})}{\sinh(\frac{\pi\omega_{in}}{\rho})\sinh(\frac{\pi\omega_{out}}{\rho})} = 0. \tag{68}$$

Thus the relation of the norm is satisfied:

$$\mid \alpha_k \mid^2 - \mid \beta_k \mid^2 = 1. \tag{69}$$

We see that the quantum field in the vacuum state $\mid 0, in >$ defined through modes u_k^{in} and u_0^{in} and u_1^{in}, when detector showed that there are no particles, was the vacuum state for non-accelerating state. This was for $\eta \to -\infty$ and Minkowski space-time was realized. In the out region $\eta \to +\infty$ we have found that the Minkowski space-time is realized again, and that the vacuum state is the same: for the inertial observer there are no particles detected in this vacuum $\mid 0, out >$ which is identical with the vacuum $\mid 0, in >$. This is the reason why we speak about generation (in expanding part of the space-time described by our conformal factor) and annihilation (in contracting period of the space-time described by our conformal factor,

in such a way that there are no particles in the limit $\eta \to +\infty$, there is only the -out vacuum identical to -in vacuum, so particles generated during expansion period up to $\eta = 0$ are annihilated in $\eta \to +\infty$) of particles due to the expanding and contracting space-time (due to gravity). In difference to the case in [10] we have found that besides the k-particles with nonzero mass there are present localized modes 0 and 1 evolving from $\eta \to -\infty$ as η is developing to $\eta \to +\infty$. So their development is present in the conformal time interval $\eta \to -\infty \leq \eta \leq \eta \to +\infty$.

Conflict of Interest.

The authors declare that they have no conflict of interest.

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